Letters

Comments on "Refraction at a Curved Dielectric Interface: Geometrical Optics Solution"

ADEL A. SALEEB

In the above paper[1] the authors solved the problem of transmission of a spherical or plane wave through an arbitrary curved dielectric interface. We use the same notation U^i and U^t for the incident and transmitted fields respectively, where

$$U = \begin{cases} E & \text{for perpendicular polarization} \\ H & \text{for parallel polarization.} \end{cases}$$

At point 2 in the second medium (Fig. 1) the fields are given by

$$U^t(2) = (DF) T e^{-jk_2 b} U^i(1). \tag{1}$$

Here T is the Fresnel transmission coefficient for a planar interface, and (DF) is the divergence factor of the transmitted ray pencil at point 2 in reference to point 1. This factor describes the cross-sectional variation of a ray pencil as it propagates in the transmitted region; it is given by

$$DF = \frac{1}{\sqrt{1+b/R_1}} \frac{1}{\sqrt{1+b/R_2}}. \tag{2}$$

R_1 and R_2 are the two principal radii of curvature of the transmitted wave front passing through point 1.

Now consider the surface shown in Fig. 2, which is a hyperboloidal surface of revolution with a point source at the focus 0. This surface converts the incident spherical wave front into a planar wave front [1]. Therefore, the transmitted wave front has principal radii of curvature $R_1 = R_2 = \infty$, and DF becomes unity. This can also be shown to be true by calculating DF through the procedure described in the paper in question. According to (1) above the transmitted fields become

$$U^t(2) = T e^{-jk_2 b} U^i(1) \tag{3}$$

which is true only for an infinite planar interface.

The actual reason behind this contradiction is that the authors of the paper in question considered only variation of the cross-sectional area of a pencil of rays as it propagates in the second medium but they neglected the sudden change in the cross-sectional area caused by refraction at the interface. To take this effect into account, (DF) must be modified as follows:

$$(DF) = (DF)_1 \cdot (DF)_2. \tag{4}$$

$(DF)_1$ is given by (2) above; $(DF)_2$ is derived below. Referring to Fig. 3, the power radiated by the source at 0 within the solid angle formed by rotating the planar angle $d\theta$ around the z axis is given by

$$2\pi P(\theta) \sin\theta \, d\theta$$

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[1] S. W. Lee *et al.*, *IEEE Trans. Microwave Theory Tech.*, vol MTT-30, pp. 12–19, Jan 1982

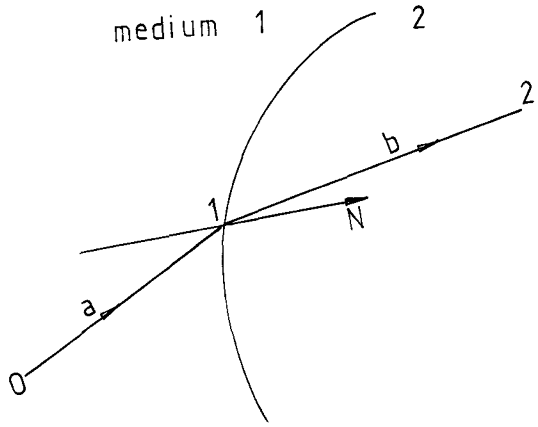

Fig. 1 Refraction at a curved dielectric interface.

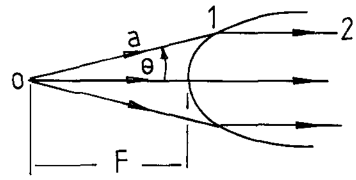

Fig 2 Hyperboloidal interface

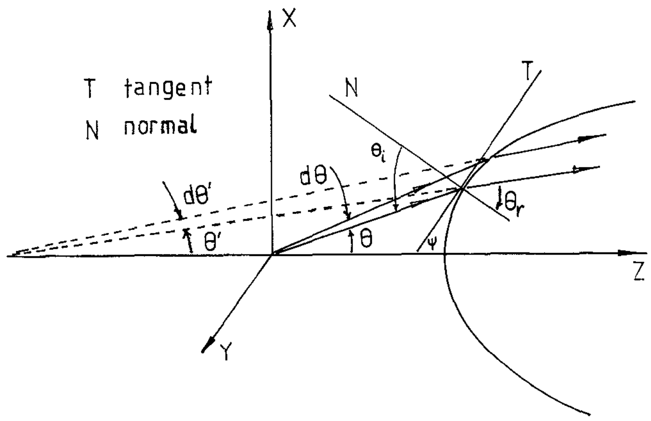

Fig 3 Geometry for calculating $(DF)_2$

where $P(\theta)$ is the radiated power intensity (power per unit solid angle). According to the principle of the conservation of energy, this power will be contained within the solid angle formed by rotating the planar angle $d\theta'$ through 360° around the z axis. Thus,

$$2\pi P(\theta) \sin\theta \, d\theta = 2\pi P'(\theta') \sin\theta' \, d\theta'$$

$$P'(\theta') = P(\theta) \frac{\sin\theta}{\sin\theta'} \frac{d\theta}{d\theta'}$$

where $P'(\theta')$ is the radiation intensity after refraction. From Fig.

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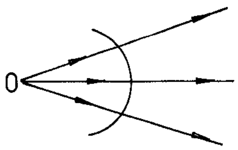

Fig 4. Concave spherical surface.

3 we can obtain the following relations:

$$\theta = \theta' + \theta_i - \theta_r$$
$$\psi = 90° + \theta - \theta_i$$
$$= \tan^{-1}\frac{dx}{dz}$$
$$r\sin\theta = r'\sin\theta'$$
$$\sin\theta_i = n\sin\theta_r \qquad \text{(Snell's law)}.$$

Thus,

$$\theta' = 90° + \psi + \sin^{-1}\left[\frac{1}{n}\cos(\theta - \psi)\right] \tag{5}$$

where n is the ratio of the refractive index of medium 2 to that of medium 1.

The radiation densities (power radiated per unit area) $S(\theta)$ and $S'(\theta')$ before and after refraction, respectively, are related to radiation intensities through

$$S(\theta) = r^2 P(\theta)$$
$$S'(\theta') = r'^2 P'(\theta').$$

Thus,

$$\frac{S'(\theta')}{S(\theta)} = \frac{r'^2}{r^2}\frac{\sin\theta}{\sin\theta'}\frac{d\theta}{d\theta'}.$$

The term $(DF)_2$ is the ratio of the electric field $U'(1)$ (just after refraction) to $U^i(1)$ (just before refraction). This ratio is given by

$$(DF)_2 = \frac{U'(1)}{U^i(1)} = \sqrt{\frac{S'(\theta')}{S(\theta)}}$$

or

$$(DF)_2 = \left(\frac{\sin\theta}{\sin\theta'}\right)^{3/2}\left(\frac{d\theta}{d\theta'}\right)^{1/2}. \tag{6}$$

The hyperboloidal surface of Fig. 2 is a special case where the angle θ' is equal to zero because of total collimation of the refracted rays and the term $(DF)_2$ can be shown to be

$$(DF)_2 \doteq \frac{(n\cos\theta - 1)^{5/2}}{F^2(n-1)^2(n-\cos\theta)^{1/2}}. \tag{7}$$

If the source is located at the center of a concave spherical surface as shown in Fig. 4, no refraction takes place because the rays are incident normally on the surface. Therefore the term $(DF)_2$ becomes unity. For surfaces other than spherical (Fig. 4) or hyperboloidal (Fig. 2) both $(DF)_1$ and $(DF)_2$ have values other than unity.

References

[1] A. A. Saleeb, "Theory and design of lens-type compact antenna ranges," Ph.D. thesis, Queen Mary College, University of London, London, U.K., July 1982

Comments on "Improved Calibration and Measurement of the Scattering Parameters of Microwave Integrated Circuits"

ROGER MARKS

The above paper[1] proposes "generalized TRL" as an alternative to the TRL and LRL calibration methods. The contributions of the work, according to the authors, are "the reformulation in terms of S parameters and the removal of the requirement to specify a line length." In fact, it appears that *only* the formulation, not the method itself, is novel.

The original TRL method [1] utilizes a zero-length through connection. A more general calibration scheme, coined LRL [2], [3], replaces the through with a transmission line. The first stage of LRL is identical to TRL; the shorter line continues to be described *mathematically* as a zero-length through. As clearly pointed out by Hoer and Engen [2], [3], this results in calibration at a pair of "mating" references planes which coincide with the center of the short line. The second stage of LRL entails the movement of the reference planes back to the physical ports. It is *only* the movement of the reference planes that requires knowledge of the line lengths.

The current proposal is apparently just the first stage of LRL. As such, it avoids the need for line lengths solely by *leaving* the references planes in the center of the short line. This is demonstrated by the equivalence of the S parameters of the proposed method (equations (28)–(31) in the paper in question) with the analogous cascade coefficients of LRL (equations (1)–(6) of [2] or [3]). The proposed calibration scheme is not an "improved" or "generalized" form of TRL except to the extent that LRL is itself a generalization of TRL.

Furthermore, the authors' claim of a new method is unsupported by their experimental evidence, which offers only a comparison between their calibration and an *uncalibrated* test fixture.

Reply[2] by R. R. Pantoja, M. J. Howes, J. R. Richardson, and R. D. Pollard[3]

The comments raise four specific points which require some explanation in order to ensure proper understanding not only of what is described in our paper but also of the whole family of calibration procedures under the increasingly common TRL classification. First we must correct a misprint in our paper. In the first paragraph of Section III-A, the symbol Δl should be l_1, the length of the shorter line.

1) It must be emphasized that what is achieved in our paper is an S-parameter formulation of the TRL/LRL algorithm and a specific *application* to MIC characterization, neither of which has previously been presented in the literature.

2) In the context of the type of measurement discussed, the main issue is to locate suitably the calibration reference planes for measurement of a MIC structure while retaining the freedom of choice for lengths of both line standards and, consequently,

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[1]R. R. Pantoja, M. J Howes, J. R. Richardson, and R. D. Pollard, *IEEE Trans. Microwave Theory Tech.*, vol 37, pp. 1675–1680, Nov. 1989.
[2]Manuscript received December 8, 1989.
[3]The authors are with the Department of Electrical and Electronic Engineering, University of Leeds, Leeds, LS2 9JT, U.K.
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